$$p_2, p_3 \approx -\frac{1}{2}\sqrt{2c+\frac{Q_{a1}}{Q_{b2}}+2\sqrt{c^2+\frac{Q_{a2}}{Q_{b2}}}} \pm \frac{1}{2}\sqrt{2c+\frac{Q_{a1}}{Q_{b2}}-2\sqrt{c^2+\frac{Q_{a2}}{Q_{b2}}}} \tag{30}$$

which is as given earlier in the paper.[1]

Comparison with Optimal Controller

It is interesting to note that the vertical channel is optimal in a larger sense. In the vertical channel, the control variables were constrained to be the simple functions of the measurement variable given in Eq. (3). Can a more complicated compensation connecting the measurement and control variables provide smaller mean-square errors? It is well known that optimal control is provided by feedback compensation that is the cascade combination of the Kalman-Bucy filter and the state-feedback optimal regulator.[3] The filter was given in Eq. (11). The regulator is

$$u = -C\hat{x} \tag{31}$$

In this problem there is no penalty associated with using large values for the control variables, so the optimal regulator gains become arbitrarily large. As a result, the estimate $\hat{x}$ is held near zero. Now it is known that under optimal control[3]

$$X = \hat{X} + P \tag{32}$$

where $X = E[xx^T]$, $\hat{X} = E[\hat{x}\hat{x}^T]$, $P = [ee^T]$. With $\hat{x}$ held near zero, the covariance of the state under optimal control is the same as the covariance of the optimal estimation error:

$$X = P \tag{33}$$

We previously established that the state of the simple closed-loop vertical channel is governed by the same differential equation, as is the estimation error of the Kalman-Bucy filter. So, for the simple vertical channel, Eq. (33) is also true. Therefore, the simple vertical channel achieves the same optimal performance as the more complicated unrestricted optimal controller.

References

[1] Widnall, W.S. and Sinha, P.K., "Optimizing the Gains of the Baro-Inertial Vertical Channel," *Journal of Guidance and Control,* Vol. 3, March-April 1980, pp. 172-178.

[2] Kwakernaak, H. and Sivan, R., *Linear Optimal Control Systems,* Wiley-Interscience, New York, 1972, p. 369.

[3] Bryson, A.E., Jr. and Ho, Y.C., *Applied Optimal Control,* Blaisdell Publishing Co., Waltham, Mass., 1969, pp. 414-419.

Comment on "Observer Stabilization of Singularly Perturbed Systems"

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IN Ref. 1 we showed that, for linear time-invariant singularly perturbed systems, a controller consisting of a linear feedback law and a Luenberger observer, both based on the reduced-order model obtained when the small parameter ϵ is set to zero, would stabilize the full-order system (and, in fact, the state estimate would converge to the appropriate part of the full system state) when ϵ is positive and sufficiently small. Although several methods of proof exist, the one used in the above note relied upon the well-known Klimushchev-Krasovskii lemma. An application of the result for autopilot-actuator design was presented.

Recently, it has come to our attention that the above result was, in fact, proved before by B. Porter[2]; however, the autopilot-actuator application does not seem to have appeared before. Although our work was quite independent of that in Ref. 2, Porter did publish the result first in 1974, and we wish to acknowledge the priority of his work.

We hope this note will help to set the historical record straight.

References

[1] Balas, M.J., "Observer Stabilization of Singularly Perturbeed System," *Journal of Guidance and Control,* Vol. 1, ~~Jan.-Feb. 1978,~~ pp. 93-95.

[2] Porter, B., "Singular Perturbation Methods in the Design of Observers and Stabilizing Feedback Controllers for Multivariable Linear Systems," *Electronics Letters,* Vol. 10, Nov. 1974, pp. 494-495.

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